IAL Activity 4 responses

P4 responses

1 Assume $\sqrt{3} = \frac{m}{n}$ with m and n natural numbers with no common factor (except 1)

$3n^2 = m^2$ and from this we can conclude that m and n are divisible by 3 so proof by contradiction.

2 We get to $4n^2 = m^2$ so m is a multiple of 2 (not necessarily 4) and there will be no contradiction.

3 This time we get to $3n^3 = m^3$ and we can show that m and n are divisible by 3 so proof by contradiction.

4 (a) Let $r = k + n$ Suppose r is rational, then $r = \frac{a}{b}$, with, $a, b \in \mathbb{N}$ and co-prime

Then $r = \frac{a}{b} \Rightarrow k + l = \frac{a}{b} \Rightarrow l = \frac{a}{b} - k = \frac{a - bk}{b}$ which is clearly rational so r is irrational.
Similarly with the product.

(b) No conclusions can be drawn. For example, $\sqrt{3} \times \sqrt{12}$ is rational $\sqrt{3} \times \sqrt{14}$ is not.

$(4 - \pi) + (3 + \pi)$ is rational.

5 Suppose it can.

$2^k = n + n + 1 + ... + n + r - 1$ where $r > 1$

Using sum of an arithmetic series $2^k = n + n + 1 + ... + n + r - 1 = \frac{r}{2}\left(2n + r - 1\right)$

$2^{k+1} = r\left(2n + r - 1\right)$ (*)

So r must be a power of 2, say 2^s $(k + 1 > s > 0)$

(*) then becomes $2^{k+1-s} = 2n + 2^s - 1$ which is a contradiction as the left had side is even but the right hand side is odd

6 $I = \int \sec^4 x\mathrm{d}x = \sec^2 x \tan x - \int \tan x 2 \sec x \sec x \tan x \mathrm{d}x =$

$I = \sec^2 x \tan x - 2\int \sec^2 x(\sec^2 x - 1)\mathrm{d}x$

$I = \sec^2 x \tan x - 2I + 2\tan x$ $I = \frac{1}{3}(\sec^2 x \tan x + 2\tan x)$ or $\frac{1}{3}\tan^3 x + \tan x + c$

7 We need a and b to be positive, so having $a, b \in \mathbb{N}$ is needed.

$2b > a > b > 1$ needs to be justified.

The final line should be completed by noting we assumed a, b in its lowest terms, but we have replaced them by numbers that are smaller, so we get a contradiction.